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Physiological Flow and Article Diseasese: a Surve

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Abstract

The Physiological Fluid dynamics are that the motion and the Present extremely interdisciplinary Physiological Flow Problems include Flow throught straight Flexible (Elastic and viscoelastic) curved branches and tapered tubs.

Keywords: Vri and Vz = Physiological Flow involves the solutions,
Pw = Density of the material,
Uz = radial and axial displacements,
P and r = isotropic Pressures.

Introduction

Physiological fluid dynamics present extremely interdisciplinary challenging problems which require for their solution Intimate cooperation between Mathematician, Engineer, Biologist and Medical scientists. Its development can have important consequences for biology and medicine on one side and for all branches of technology on the other side. By nature, of physiological system or a unit is so optimally designed that it is almost impossible to make any improvement in the system itself but knowledge and clear understanding of the system and its function will be helpful in constructing suitable models for diagnosis and designing designing artificial units or organs for replacement [Lighthill (1975)]

The main characteristics of the physiological fluid dyanamics are that the motion of the fluid is energised by the mobility of the animals external or internal flexible surfaces. Apart from the well-known physiological fluids: air, water and blood, there are many others like urine, tears, bile cerebrospinal and synovial fluids. While air, urine, sweat, tears may be treated as Newtonian fluids, blood and synovial fluid exhibit non Newtonian characteristics.blood is the most common biological fluid representing the characteristics of almost all the physiological fluidsunder different physical and biological environment and therefore it has been studied in greater details from various points of view [Chmiel (1974), Thurston (1972), Phillips and Deutch (1975)]

Aim of the Study

The human circulatory system consists of s complex net work of blood vessels and performs important functions; Respiratory, Nutritive, Excretory, Protective and Regulatory in human body. The internal diameters of the blood vessels range from 2.5 cm to 4 mocrons ($1\mu.m = 10^{-6}$m.). Most of the efforts in physiological flows have been directed towards the problems related to circulatory and respiratory system.

Mathematical Modelling of the Flow Problems in General

The physiological flow problems include flow through straight, flexible (elastic and viscoelastic) curved branchesand tapered tubes with or without local constructions. Governing equations for two dimensional axi symmetric in compressible fluid flow systems may be written from standard text [Bird, Stewart and Lightfoot (1962)].

$$\rho\left(\frac{\partial v_r}{\partial t}+ v_r\frac{\partial v_r}{\partial t}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial r}+\frac{\partial \sigma_{rr}}{\partial r}+\frac{\partial \sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} \quad (1)$$

$$\rho\left(\frac{\partial v_z}{\partial t}+ v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}+\frac{\partial \sigma_{zz}}{\partial z}+\frac{\partial \sigma_{rz}}{\partial r}+\frac{\sigma_{rz}}{r} \quad (2)$$

$$\frac{\partial v_z}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = 0 \quad (3)$$

These are the basic governing equations in the variables V_{r1}andV_z Physiological flows involves the solution of these equations for a well defined fluid system represented by the constitutive equations for the flute subjects to the continuity of the stress and velocity field at the inner surface of the vessel walls along with the symmetry of the velocity field on the axis and prescribed stress field at the count outer boundary of the vessel neglecting the internal effects governing equations for wall may be written in the following form

$$\rho_w \frac{\partial^2 u_r}{\partial t^2} = \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} - \frac{\partial \Omega}{\partial r} \quad (4)$$

$$\rho_w \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} - \frac{\partial \Omega}{\partial z} \quad (5)$$

$$\frac{\partial u_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r u_r) = 0 \quad (6)$$

ρ_w is the density of the material of the wall u_r and u_z are the radial and axial displacements. Thus the problem is to solve the system of the equations with prescribe constitutive relations for the fluid and the material of the wall subject to the following conditions:

(A) Symmetry of the velocity field at the axis of the container

$$v_r = 0 \quad \frac{\partial v_z}{\partial r} = 0 \quad at \quad r = 0 \quad (7)$$

(B) Continuity of the velocity field at the fluid/solid interface :

$$v_r = \frac{\partial u_r}{\partial t} v_z = \frac{\partial u_z}{\partial t} \quad at \quad r = a \quad (8)$$

(C) Continuity of the share and radial stresses at the fluid/solid interface :

$$\sigma_{rz} = \tau_{rz}$$
$$-P + \sigma_{rr} = -\Omega + \tau_{rr} \quad at \quad r = a \quad (9)$$

(D) Prescribed express field at the outer surface:

$$\tau_{rz} = 0$$
$$-\Omega + \tau_{rr} = 0 \quad at \quad r = b (10)$$

In these equations p and Ω are the isotropic pressures in the fluid and the elastic walls. Although the vessel walls are much more complex consisting of five different layers : Endothelial cells, Intima, Elastin, Media and Advantita and each describes altogether difference material properties and cannot be represented by overall elastic or viscoelastic meterials. Experience in the solutions of these equations may be useful if one is ultimately interested in more realistic models.

Analytical solution for pulsatile flows forNewtonian fluids were obtained by Womerseley (1955) for unconstrained and elastic tube and later on in a subsequence paper (1957)he included longitudinal constraintsLetter on Attabeck and Lew (1966) and Attabeck(1968) extended Womerseley's work for excel and circumferential stresses and included and property also.Cox (1968) introduced visoelastic tubes also. Taylor (1955, 1966) extended the plusatile flow problems for branching. Several researchers where typing these problems by adding complexities one after another including various properties of the fluid as well as balls solutions procedures for numerical treatment of two and three-dimensional rectangular coordinates with prescribed boundary conditions on the wall in the form of thin membrane in have been described by Sikh 1968 and rocche 1975 20 may lead to further development of numerical methods for physiological flow problems and some eltron 1978.

Arterial Diseases

The human arterial system is not just a transmission line for blood and other materials but it is an organ which is permeated by various chemicals within which important metabolic processes occur. It there occurs a slight change in overall optimal conditions in a particular organ or unit, serious diseases crop in and further complications may prove to be fatal. For example, lipid metabolism and transport through the arterial wall plays a key role in the development of theinner wall disease "Atherosclerosis". If enzyme failure occurs, the cholesterol in take through fatty diets will not be broken into simpler product and it will deposit locally in the arterial intima of large molecules to the tissue adds to inner layer disease. Therefore the study of diffusion and filtration across the membrane and through the tissues would be necessary to further understanding of Atherosclerosis. Thus, for our further analysis, we include the diffusion equation also [Lightfoot (1974), Tandon (1979)].

$$\frac{DC_1}{Dt} + \nabla . J_1 + \rho_1 = 0 \quad (11)$$

Where $C_1$and $J_1$and $\rho_1$are the concentrations, flux and the rate of production of Species within the system. Several approaches to solutions appeared recently [Ulanowicz and Frazier (1970)]

Transport of certain materials between interluminal blood and arterial wall appears to be deposted at low shear stress regions [Middleman (1972) and Narem (1973)]. Therefore, a knowledge of the wall shear stress in an important factor for many arterial diseases. For example, very high shear and normal stresses damage the erythrocytes which are filtered through the spleen – a process which in medical terminology is called 'hemolysis.

Local Vessel Constrictions

Arteries may be locally constricted by the deposition of intravascular plaques which grow inwards from the wall and the diseas larg enough to produce separated flow regions the possibility of more rapid growth could be introduced by the separated flows, Further, the possibility of thrombus formation in such slowly recirculating regions also exists. The weakening and buldging of the artery downstream from stestenosis is another major complication of this occlusive vascular disease. In order to locate very high and very low shear regions during the formation of stenosis, the solution of the primitive equations under the idelised fixed or time dependent growing stenosis is necessary. Analytical solution for the very restricted class of problems has been obtained by Morgan and Young (1974) for the approximate solutions for velocity and stress distributions in a fixed idealised stenosis. Mac Donald (1978) and Dooren (1978) have obtained analytical solution for steady flows in tubes of slowly varying and constricted or widened tubes respectively. Tandon et al (1978) have extended the problem for time dependent growing stenosis defined by the radius of the artery in the stenotic region as

$$R = R_v - a_v \left(1 - e^{\frac{-t}{T}}\right)\left(1 + \cos\frac{\pi z}{z_v}\right) \quad (12)$$

Later on, Tandon et al (1979) also discussed the flow through long tube of slowly varying radius and discussed the effects of reabsorption in decreasing the wall shear stresses. The problem is important for determining the glomerular filterate among a population of nephrons in renal circulation. With the knowdedge of the fraction of the filterate which has been reabsorbed in passing through a known lenth of tube, a relation representing the variations of shear stress on the wall with axial coordinates can play a significant role.

Survey of the Numerical Treatment of the Flows through Constricted Channels

The topic itself is suitable for further theoretical analysis for the challenging problems referred above. The principal difficulties involved in applying numerical methods to physiological flows of circulatory and or respiratory systems involving destensible boundaries include.

1. Approximating the irregular wall shape.
2. Accounting for the non-Newtonian character of the physiological fluids.
3. Duplication of the pulsatile flows.

One of the most efficient methods of dealing with the irregularly shaped boundaries is the use of coordinate transformation Lee and Fung(1970) used conformal mapping to transform a bell-shaped local constriction to an infinity long constant diameter tube and obtained numerical solutions for wall shear stresses. Using non-orthogonal transformation, Oberkambf and Goh (1974) transformed the same bell-shaped local constriction into a constant diameter tube of finite length. A recent method for the reneralized domain mappings for the first and non-conservative system has been developed by Oberkampf (1976). A procedure for calculating velocity and shear fields at the bifurcations has been developed by Lynn et al (1970). Ray and Davids (1970) have studied flow through artificially creacted local shear rates and concluded that the obstructions enhance local shear rates and produce distortion in flow field. Immediately past an obstruction, the shearing stress falls much below the unobstructed value.

All these aspects of the physiological flows are related to the solutions of the primitive equations (1 to 3) under different physiological circumstances. Eliminating p between (1) and (2) for the case of Newtonian fluids only and introducing the vorticity relation:

$$\xi = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \quad (13)$$

The following vorticity transport eduation may be obtained.

$$\frac{\partial \xi}{\partial t} + \frac{\partial}{\partial r}(v_r \xi) + \frac{\partial}{\partial z}(v_z \xi) = v\left(\frac{\partial^2 \xi}{\partial z^2} + \frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r}\frac{\partial \xi}{\partial r} - \frac{\xi}{r^2}\right)$$
(14)

Again, introducing the stream function defined by

$$\frac{1}{r}\frac{\partial \varphi}{\partial r} = v_z \quad and \quad \frac{1}{r}\frac{\partial \varphi}{\partial z} = -v_r \quad (15)$$

We define vorticity in terms of φ as

$$-\xi = \frac{1}{r}\left(\frac{\partial^2 \varphi}{\partial r^2} - \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2}\right) \quad (16)$$

Pressure distribution may be obtained from the Poisson;s equation as

$$\frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial z^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{2}{r^2}\left[\frac{\partial^2 \varphi}{\partial z^2}\left(\frac{\partial^2 \varphi}{\partial r^2} - \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - \left(\frac{\partial \varphi}{\partial z}\right)^2 + \right.$$
$$\partial 2\varphi\partial z\partial r 1 r\partial\varphi\partial r - \partial 2\varphi\partial z\partial r \quad (17)$$

Further, the introduction of ξ from (16) into (14), a biharmonic equation in may be obtained

$$\frac{\partial}{\partial t}(D^2\varphi) - \frac{1}{r}\frac{\partial(\varphi, D^2\varphi)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial \varphi}{\partial r}\, D^2\varphi = vD^4\varphi \quad (18)$$

Where $D^2 \equiv \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\ and\ D^4\varphi = D^2(D^2\varphi)$
(19)

The equation (18) is a non-linear fourth order differential equation and one should make all efforts to solva this equation satisfying initial and boundary conditions. Roache (1975) has suggested an efficient method for solving steady planner biharmonic equation

$$\frac{\partial \varphi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\varphi) - \frac{\partial \varphi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\varphi) = v\nabla^4\varphi \quad (20)$$

The numerical solutions which have been obtained for such problems till date must be considered as a foundation for further study. There is a majority of numerical solutions for two dimensional flows through a simple boundary [Roache and Muller (1970), Daly (1975)].

In all these attempts, an initial velocity profile is always assumed, whereas the problem of blood flow is altogether different from any of the fluid dynamical problems. There is no such thing like an initial velocity for blood flows in aorta andtension – versus stretch relation for the wall cannot be measured from such flows. The velocity itself cannot be measured directly, although it can be computed from other measurements. Therefore, some analysis based on physical measurementsis necessary for physically fruitful results. Recently, Leberstein (1974) has proposed a scheme based on mean square asymptotic instability in giving a unique solution inspite of the impossibility to specify an initial velocity profile. The assumption made is many fewer, more precise and much less drastic than those used elsewhere in the treatment of blood flows. The technique of ignoring initial velocity for viscous flows and looking at ‘steady state’ development in mean square asymptotic uniquencess is one that deserves general attention.

Further Scope

In the problems of interest, extensive viscous separated flow regimes occur and one must look for complete mathematical models. Pathogenic effects of turbulence have been clearly demonstrated by Roache (1963a. 1963b) and she discussed the influence of turbulrnce in post stenotic dilation, Separated flow regimes and the stagnant regions in post-stenotic dilatations are other areas of interest to numerical analysts weakening of the arterial walls in response to fluctuating flows associated by the turbulence is the self-aggrevating pathological condition with serious consequences.

Conclusion

Intracranial arteries have smaller ratio of the wall thickness to diameter with lower elastic constants and they are not well supported by the surrounding tissues. Under the conditions of high cardiac output, the radius of the artery grows sufficiently large with an associated reduction in the minimum thickness resulting to rupture leading to cerebral Haemorrage.Thus, the turbulence occurring at low shear regions in post stenotic dilatation and at the bifuecations plays a dominant rolw in the development of turbulence [Davies (1972)] has further scope for locating the sites right at the very transitional state because the turbulent stresses are two to three orders of magnitude greater than in normal laminar flows [Ferguson (1972)].

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